

Maths Notes

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Discrete Mathematics



Unit - 1

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Set: A Set is a well-defined collection of distinct objects (i.e. elements).

In another words, A set is a collection of well-defined objects or elements that are unique and don't change from person to person.

Example:- Set of Natural Numbers

$$N = \{1, 2, 3, \dots\}$$

Set of whole Number

$$W = \{0, 1, 2, 3, \dots\}$$

Key Properties of Sets:

1. **Distinct Elements:** Each element in a set is unique. A Set does not contain Duplicate elements.
2. **Unordered:** The Order of elements in a set does not matter. for example, the sets $\{1, 2, 3\}$ and $\{3, 2, 1\}$ are considered the same.

Difference Between Roaster form and Set Builder form

Roaster Form (Tabular Form):

Definition: The Roaster form of a set specifies all its elements explicitly by listing them between curly braces. i.e. no. of elements surrounded by curly braces and separated each element by comma.

example: $\{1, 2, 3, 4\}$ $\{5, 6, 8, 9\}$

I - fin U

Example: If we have a set of natural numbers less than 5, it can be written as $\{1, 2, 3, 4\}$.

Usage: This form is straightforward and used when the set has a relatively small and well-defined number of elements.

Notation: Elements are separated by commas and listed inside curly braces.

Advantages: Easy to Read and Understand for small sets.

DisAdvantages: Not Practical for large sets or sets defined by a specific property.

Set-Builder Form:

Definition: The Set-Builder form of a set specifies the properties that its elements must satisfy.

Example: The Set of Natural Numbers less than 5 can be written as $\{x \mid x < 5, x \in \mathbb{N}\}$

where, x such that, $x \in$ belongs to, $\mathbb{N}$ = no. of all Natural Numbers

Usage: This form is useful for describing sets with a large or infinite number of elements, or when the set is defined by a specific condition.

Notation: Typically involves a variable (such as x), a vertical bar or colon ($\mid$ or $:$), and a condition that describes the elements of the set.

Advantages: Concise for large or infinite sets, and provides a ~~set~~ clear sub for member-ship.

Disadvantage: Can be less intuitive or more complex to understand compared to the roster form.

Both Examples:

• Roster Form: Set of Vowels: $\{a, e, i, o, u\}$
Set of prime numbers less than 10: $\{2, 3, 5, 7\}$

• Set-Builder Form:

Set of even numbers: $\{x | x \text{ is an even integer}\}$

Set of numbers greater than 2: $\{x | x > 2\}$

Both Readability:

• Roster Form: More readable for small sets.

• Set-Builder Form: Can be more abstract and requires understanding the condition defining the set.

Types of Set:

- ① Finite Set: Finite sets are sets with a finite or countable number of elements. These sets are also known as countable set because their elements can be counted. In other words, A set with a limited number of elements.

Example: $\{1, 2, 3, 4\}$ or $B = \{a, e, i, o, u\}$

Cardinality of Finite Set

The number of elements in a finite set is known as the cardinality of that set. The cardinality of any finite set A is represented by $n(A)$.

example:

Set $A = \{1, 2, 3, 4, 5\}$ Cardinality of set A is $n(A) = 5$

Properties of Finite Sets

The following finite set conditions are always finite.

- ① Finite Sets have a definite count of elements (Cardinality)
- ② Every subset of a finite set is also finite.
- ③ The Union and intersection of finite sets are finite
- ④ Empty Set is a finite set with no elements.
- ⑤ Power set of a finite set is also finite.
- ⑥ Cartesian product of finite sets is finite.

② Singleton Set: A Set is said to be singleton set when the number of elements in the set is one. The set of the form $S = \{a\}$ is called a singleton set. The Singleton sets are always finite as it has only 1 element.
 Singleton Set is defined as a set containing only one element

Singleton Set Symbol: $P = \{x\}$

where, P is a singleton set with an element x .

Singleton Set examples: All sets with only one element are singleton sets. example of singleton set are,

- Set of even prime numbers
- Set $B = \{10\}$
- Set $C = \{1/x : x^2 = 16\}$

Cardinality of Singleton Set: The Cardinality of Singleton Set is 1 as the number of elements present in singleton set is 1. [Cardinality of Singleton Set = 1]

Power Set of Singleton Set contains two elements the null set and singleton set. Therefore Cardinality of ^{Power Set of} Singleton Set is 2

Properties of Singleton Set:

- ① Singleton Set has only 1 element.
- ② Singleton Set has the cardinality one.
- ③ The Cardinality of power set of Singleton Set is 2.
- ④ Empty Set is subset of every Singleton Set.
- ⑤ The Union of two Singleton Set is Singleton when two sets are

- ③ Well-Defined Set: A Set is a well defined if there is no ambiguity as to whether, or not an object belongs to it, i.e., a set is defined so that that we can always tell what is and what is not a member of the Set.

Example: $C = \{\text{red, blue, yellow, green, purple}\}$ is well defined since it is clear what is in the set.

- ④ Null Set / Empty Set / Void Set: Empty Sets are sets with no items or elements in them. They are also called null sets. The Symbol ϕ (phi) represents the empty set and is written as ϕ , It is also denoted by $\{\}$. When compared to other sets, Empty Sets are seen to be Distinctive.

The Empty Set is a set that contains no elements. It is a unique set that is defined by the absence of any members; Empty Set Symbol ϕ (phi) @ $\{\}$

Properties of Empty Set: ① Empty Set is a Subset of Every Set.

② Universal Subset: The Empty set is a subset of every set.

③ Uniqueness: There is only one empty set.

④ Cardinality: The Empty Set has a cardinality of 0.

⑤ Intersection: The Intersection of any set with the empty set is the empty set itself.

- ⑤ Infinite Set: The Uncountable Element or Item in Set is called Infinite Set. All Sets that contain either an uncountable number of elements or if countable then an unlimited number of elements are called infinite Sets. One of the Most Common Examples of an Infinite Set is a Set of all natural numbers i.e. $\mathbb{N}$.



The Infinite Set is Denoted by using dots ($\dots$)..

i.e. $A = \{a_1, a_2, a_3, \dots\}$

★ Infinite Set Examples

① Natural Numbers: Set of all Natural Numbers

$$\mathbb{N} = \{1, 2, 3, 4, \dots\}$$

② Integer: Set of Integers ($\mathbb{Z}$)

$$\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

★ Cardinality of Set: The Infinite Set has not be countable so the cardinality is infinite.

⑥ Equivalent Set: An Equivalent Set refers to two sets that have the same number of elements, or the same cardinality, regardless of what the elements are.

This means that for each element in one set, there is a corresponding element in the other set, but the specific elements themselves do not have to be the same.

Example: let Set $A = \{1, 2, 3\}$

$$\text{Set } B = \{a, b, c\}$$

Both Sets A and B have 3 elements, so they equivalent sets, even though the elements in each set are different.

⑦ Equal Sets: Equal Sets are those sets whose cardinality is the same and whose all elements are equal. In other words, two sets are regarded as equal sets when they have all the same elements and also the same number of elements.

If all elements of two or more sets are equal and the number of elements is also equal, then the sets are said to be equal sets.

Example: If $P = \{a, b, c, d\}$ & $Q = \{a, b, c, d\}$ are equal sets since they both have the same elements and also same number of elements.

Equal Set Symbol: Equal Set Represented by " $=$ ".

$$\{2, 3, 5\} = \{3, 2, 5\}$$

unequal set is Represented by

$$\{2, 3, 5\} \neq \{1, 2, 3\}$$

⑧ Subsets: A Set 'A' is a subset of Set 'B' if all the elements of Set A comes under set B. Also, a subset can be equal to a set in a particular case when all the elements of a subset are contained in the set.

① For example: Let Set A contains {apple, banana} and Set B contains {all fruits} then A is a Subset of B.

② Let Set A contains {a, e, i, o, u} and Set B contains {All alphabets} then A is a subset of B.

③ Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 4\}$ therefore B is a Subset of A.

The Subset is denoted by $\subseteq$.
example ①, ②, ③ $\Rightarrow A \subseteq B$

⑨ Super Set: A Super set is a set that contains all the elements of another set. If Set B is a superset of set A, then every element in A is also in B. This Relation-ship is denoted by $B \supseteq A$.



Example: Set • Set $A = \{1, 2\}$
and Set $B = \{1, 2, 3\}$

Here, B is a superset of A because all elements of A (which are 1 and 2) are also in B .

If B is a superset of A and $B \neq A$ (i.e., B has more elements than A), then B is called a proper superset of A . This is denoted by $B \supset A$.

★ Properties of Superset

- ① Every Set is a superset of itself.
- ② Each Set is a Subset of itself.
- ③ Because the null set includes no items, we may claim that any set is a superset of an empty set for example, every set H would be represented as $H \supset \phi$.

⑩ Proper Subset: A Set A is said to be a proper subset of set B if A is a subset of B / Set B and is not equal to B .

It is Denoted By $\subset$. (पूरा नहीं होता)

example:-

$$B = \{1, 2, 3, 4, 5\}$$

$$A = \{1, 2, 3, 4\}$$

$\therefore A$ is not equal to B .

$\therefore A$ is a proper subset of B .
 $A \subset B$.

- ② **Universal Set:** The Universal Set is the Set that contains all the objects or elements under consideration in a particular discussion or problem. It is usually denoted by U . The Universal Set is the "universe" of discourse, meaning it includes everything relevant to the content. Every other set in this content is considered a subset of the universal set.

Example: Suppose we are discussing the set of all students in a school. The Universal set U could be defined as the set of all students in that school.

Let $U = \{ \text{All student in the school} \}$

Let $A = \{ \text{Students in Science Department} \}$

Let $B = \{ \text{Students in Commerce Department} \}$

Here

- $A \subseteq U$ (The Science Department is the Subset of all Students.)

- $B \subseteq U$ (The Commerce Department is the Subset of all Students.)

The complement of A , denoted by A' , would be the set of students who are not in the Science Department, i.e., $A' = U - A$.

Example: Let

$U = \{ \text{Set of Integers} \}$

$A = \{ \text{Set of Even Integers} \}$

$B = \{ \text{Set of Odd Integers} \}$

$\therefore A$ and B is the Subset of Universal Set U .

Here, A' (the complement of even Integers) would be the Set of odd Integers, so $A' = B$.



(12) **Power Set**: The Power set of a given Set S is the set of all possible subsets of S , including the empty set ϕ and the Set S itself. The Power Set is denoted by $p(S)$.

example: Let consider a simple Set $S = \{a, b\}$

The power set $p(S)$ will be.

$$p(S) = \{ \{a\}, \{b\}, \{a, b\}, \phi \}$$

It contains

- The Empty Set: ϕ
- The Subsets containing one element: $\{a\}, \{b\}$
- The Subset containing both elements: $\{a, b\}$

$$\text{The power set } p(S) = \{ \{a\}, \{b\}, \{a, b\}, \phi \}$$

The Power Set have an formula that find no. of elements in it. The formula is 2^n where n is the no. of element in original set.

example: $S = \{a, b\}$

In S set has two elements $\therefore$ no. of elements in S set is 2, It means in power set of Set S has 2^2 elements or 4 elements in power set of S .

Operations on Sets

(1) **Union of Two Sets**: The Union of two Sets is the set that contains all the elements from both Sets, with no duplicates. If you have two sets, say A and B , the union of these sets is written as $A \cup B$.



Let S is the Set
 containing the empty
 set is

$\{a, b\}$

$\{, \emptyset\}$

Set: $\{a\}, \{b\}$

Set: $\{a, b\}$

Set: $\{a, b\}, \emptyset\}$

Set find n

is 2
 original set.

of elements in
 the set of
 elements.

Set is the
 both sets,
 sets,
 sets is

Example: Let $A = \{1, 2, 3\}$

$B = \{3, 4, 5\}$

The Union of A and B are:

$$A \cup B = \{1, 2, 3, 4, 5\}$$

Important Points:

★ The Union includes all unique elements from both sets.

★ If an element is in both sets, it is included only once in the union.

② Intersection of Two Sets: The Intersection of two sets is the set that contains only the elements that are present in both sets. If you have two sets, let A and B , the Intersection of these sets is written as $A \cap B$.

Intersection symbol is denoted by $\cap$.

Example: Let $A = \{1, 2, 3\}$

Let $B = \{3, 4, 5\}$

The Intersection of A and B is

$$A \cap B = \{3\}$$

Important Points

★ The Intersection includes only the elements that are common to both sets.

★ If there are no common elements, the intersection is an empty set $\emptyset$.

③ Disjoint of Two Sets: Two Sets are said to be Disjoint if they have no elements in common. In other words, the intersection of two disjoint sets is the empty set.



Example: Let $A = \{1, 2, 3\}$

Let $B = \{4, 5, 6\}$

Since there are no elements common to both A and B, these sets are Disjoint. In another and simple words, A is disjoint of B.

$$A \cap B = \phi$$

Important Points

- ★ Disjoint Sets have an empty Intersection.
- ★ If A and B are disjoint, then $A \cap B = \phi$.

(4) Difference of Two Sets: The Difference of Two Sets, often called the Set Difference, is the set of elements that are in one set but not in the other. If you have two sets A and B, the Difference of A and B is written as $A - B$ or $A \setminus B$.

Example: Let $A = \{1, 2, 3, 4\}$
Let $B = \{3, 4, 5, 6\}$

The Difference of A and B (i.e., $A - B$) is the set of elements in A that are not in B:

$$A - B = \{1, 2\}$$

Similarly, the Difference of B and A (i.e., $B - A$) is the set of elements in B that are not in A:

$$B - A = \{5, 6\}$$

★ Important Points

- ★ $A - B$ contains elements that are in A but not in B.
- ★ The Difference operation is not commutative, meaning $A - B$ is generally different from $B - A$.
 $A - B \neq B - A$.

⑤ Symmetric Difference Between Two Sets?

The Symmetric Difference Between Two Sets is the set of elements that are in either of the sets, but not in their Intersection. In other words, it's the set of elements that belong to one of the sets but not both.

(Generally denoted by Δ).

Let If you have two sets A and B, the symmetric difference is denoted by $A \Delta B$ and is defined as:

$$A \Delta B = (A - B) \cup (B - A)$$

where:

$A - B$ is the Set of elements in A but not in B.

$B - A$ is the ~~set~~ set of elements in B but not in A.

Example:

Let's say: $A = \{1, 2, 3\}$

$B = \{3, 4, 5\}$

The Symmetric Difference $A \Delta B$ would be:

- $A - B = \{1, 2\}$ (elements in A but not in B)
- $B - A = \{4, 5\}$ (elements in B but not in A)

$$\text{So, } A \Delta B = \{1, 2, 4, 5\}$$

⑥ Complement of a Set: The Complement of a Set refers to the elements that are not in the given set but are in the universal set. The Universal Set is the set that contains all the possible elements under consideration, typically denoted by U.



If A is a Set and U is a Universal Set, the Complement of A , denoted by A^c or $\bar{A}$, is defined as:

$$A^c = \{x \in U \mid x \notin A\}$$

Example: Let The Universal Set $U = \{1, 2, 3, 4, 5\}$

The Set $A = \{2, 3\}$

The Complement of A is: $A^c = \{1, 4, 5\}$

These are the elements in U that are not in A .

Important Points

- ★ The Complement of the Universal Set U is the empty set ϕ , because there are no elements outside the universal set.
- ★ The Complement of the Universal Set is the Empty Set, since all elements are outside the empty set.
- ★ The Operation of taking a complement is related to the idea of negation in Logic. If A represents a condition, then A^c represents the negation of that condition.

Laws

- ① Idempotent Law: $A \cup A = A$
 $A \cap A = A$
- ② Identity Law: $A \cup \phi = A$
 $A \cap \phi = \phi$

③ Commutative Law: $A \cup B = B \cup A$
 $A \cap B = B \cap A$

④ Associative Law: $(A \cup B) \cup C = A \cup (B \cup C)$
 $(A \cap B) \cap C = A \cap (B \cap C)$

⑤ Distributive Law: ① $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
 ② $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

⑥ De Morgan's Law:
 ① $(A \cup B)' = A' \cap B'$
 ② $(A \cap B)' = A' \cup B'$

★ Important Results (Inclusion/Exclusion)

① $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

where, n is no. of elements.

$n(A)$ = no. of element in Set A.

$n(B)$ = no. of element in Set B.

$n(A \cap B)$ = no. of element in Set $(A \cap B)$.

$n(A \cup B)$ = no. of element in Set $(A \cup B)$.

② $n(A \cup B \cup C) = n(A) + n(B) + n(C)$
 $- n(A \cap B) - n(B \cap C)$
 $- n(A \cap C) + n(A \cap B \cap C)$

Same, —||—

Q=① In a Group of 500 Students there are 475 Students who can speak Hindi and 200 can speak English. What is the number of Students



can Speak Hindi Only?

Solve $\Rightarrow$

$$n(H \cup E) = 500$$

Let no. of Students who can speak Hindi
 $= n(H)$

no. of Students who can speak English
 $= n(E)$

A.T.Q. (According to Question)

$$n(H \cup E) = 500$$

$$n(H) = 475, n(E) = 200$$

$$n(H \cup E) = n(H) + n(E) - n(H \cap E)$$

$$500 = 475 + 200 - n(H \cap E)$$

$$500 = 675 - n(H \cap E)$$

$$-675 + 500 = -n(H \cap E)$$

$$-175 = -n(H \cap E)$$

$$n(H \cap E) = 175$$

~~175 Students can speak Hindi only~~

So the Number of Students who can
 speak Hindi Only $= 475 - 175$

$$= 300$$

$\therefore 300$ Students speak Hindi Only.

Q = Q From 50 Students Taking Examination in
 Mathematics, Physics and Chemistry each of this
 student has passed in atleast one of the Subject,
 37 passed in mathematics, 24 physics, 43 Chemistry

Atmost 19 passed mathematics and physics, atmost 29 mathematics and chemistry atmost 20 physics and chemistry. What is the largest possible number that could have passed all three examination.

To solve this principle we will use principle of Inclusion - Exclusion and the Information Given.

- Total Number of Students $n(U) = 50$
- No. of Students who passed in Mathematics $n(M) = 37$
- No. of Students who passed in Physics $n(P) = 24$
- No. of Students who passed in Chemistry $n(C) = 43$
- Maximum No. of Students who passed both Mathematics and physics $n(M \cap P) \leq 19$
- Maximum No. of Students who passed both Mathematics and Chemistry $n(M \cap C) \leq 29$
- Maximum No. of Students who passed both Physics and Chemistry $n(P \cap C) \leq 20$

We need to find largest possible number of students who have passed all three examinations, i.e., $n(M \cap P \cap C)$.

Applying the Inclusion-Exclusion Principle:

$$n(M \cup P \cup C) = n(M) + n(P) + n(C) - n(M \cap P) - n(M \cap C) - n(P \cap C) + n(M \cap P \cap C)$$

Since All Students Passed at least one Subject,

$$n(M \cup P \cup C) = n(U) = 50 \quad (\text{Substituting Value})$$

$$50 = 37 + 24 + 43 - n(M \cap P) - n(P \cap C) - n(M \cap C) + n(M \cap P \cap C)$$



Simplifying,

$$50 = 104 - (n(MNP) + n(MNC) + n(PNC)) + n(MNPNC)$$

$$n(MNPNC) = n(MNP) + n(MNC) + n(PNC) - 54$$

$$n(MNPNC) = 19 + 29 + 20 - 54$$

$$n(MNPNC) = 14$$

∴ The Largest Possible Number of Students that have passed all these examinations is 14.

Q = For All Sets A, B and C

$$If (A \cap B) \cup C = A \cap (B \cup C) ?$$

Justify your statement

Solⁿ ⇒ Let $A = \{1, 2, 3\}$ $B = \{2, 3, 4\}$
 $C = \{4, 5, 6\}$

Now

$$A \cap B = \{2, 3\} \quad B \cup C = \{2, 3, 4, 5, 6\}$$

by L.H.S.

$$(A \cap B) \cup C = \{2, 3\} \cup \{2, 3, 4, 5, 6\}$$

$$(A \cap B) \cup C = \{2, 3, 4, 5, 6\} \quad \text{--- (1)}$$

by R.H.S.

$$A \cap (B \cup C) = \{1, 2, 3\} \cap \{2, 3, 4, 5, 6\}$$



$$A \cap (B \cup C) = \{2, 3\} \quad \text{--- (2)}$$

By Equations (1) and (2)

$$\therefore \text{L.H.S.} \neq \text{R.H.S.}$$

$$\therefore (A \cap B) \cup C \neq A \cap (B \cup C)$$

Proved
✓

Q = State and Prove De Morgan's Law

$$(1) (A \cup B)' = A' \cap B'$$

$$(2) (A \cap B)' = A' \cup B'$$

Let's prove (1): $(A \cup B)' = A' \cap B'$

$$\text{Let } x \in (A \cup B)' \quad \text{--- (1)}$$

$$x \notin (A \cup B)$$

$$\Rightarrow x \notin A \text{ or } x \notin B$$

(does not belong to)

$$\Rightarrow x \in A' \text{ and } x \in B'$$

(Let and $\cap$ or $\cup$)

$$\Rightarrow x \in (A' \cap B') \quad \text{--- (2)}$$

from eq (1) and (2)

$$(A \cup B)' \subseteq A' \cap B' \quad \text{--- (A)}$$

$$\text{Let } y \in (A' \cap B') \quad \text{--- (3)}$$

$$y \in A' \text{ and } y \in B'$$

$$\Rightarrow y \notin A \text{ and } y \notin B$$

$$\Rightarrow y \in (A \cup B)' \quad \text{--- (1)}$$

$$(A' \cap B') \subseteq (A \cup B)' \text{ ————— (B)}$$

Proved De Morgan's Law.

① Statement: The Complement of the union of the two sets A and B will be equal to the intersection of A' (Complement of A) and B' (Complement of B). This is also known as De Morgan's Law of Union. It can be represented as $(A \cup B)' = A' \cap B'$.

Statement : The complement of the intersection of A and B will be equal to the union of A' and B' . This condition is called De Morgan's Law of Intersection. It can be given by $(A \cap B)' = A' \cup B'$.

$$(A \cap B)' = A' \cup B'$$

Proof ②: $(A \cap B)' = A' \cup B'$

~~Let~~ $x \in A \cap B$ ①

$\Rightarrow n \in A$ and $n \in B$ let n and U or

$$\Rightarrow x \in A' \text{ or } x \in B'$$

$$\Rightarrow x \in (A' \cup B') \text{ ————— (3)}$$

from equation (1) and (2)

$$(A \cap B)' \subseteq A' \cup B' \text{ ————— (A)}$$

$$\text{let } y \in A' \cup B' \text{ ————— (3)}$$

$$\Rightarrow y \in A' \text{ or } y \in B'$$

$$\Rightarrow y \notin A \text{ and } y \notin B$$

$$\Rightarrow y \notin (A \cap B) \text{ ————— (4)}$$

$$\Rightarrow y \in (A \cap B)' \text{ ————— (4)}$$

from equation (3) and (4)

$$(A' \cup B') \subseteq (A \cap B)' \text{ ————— (B)}$$

Hence from equation (A) and (B)
Proved De Morgan's Law

Q. Use Properties of Sets to Prove that for all the Sets A and B

$$A - (A \cap B) = A - B$$

Proof- we have $A - (A \cap B)$

$$\{ \because A - B = A \cap B' \}$$

$$\Rightarrow A \cap (A \cap B)'$$

$$\left\{ \begin{array}{l} \text{From De Morgan's Law} \\ (A \cap B)' = A' \cup B' \end{array} \right\}$$

$$\Rightarrow A \cap (A' \cup B')$$

(By Distributive Law)

$$\Rightarrow (A \cap A') \cup (A \cap B') \quad \{A \cap A' = \emptyset\}$$

$$\Rightarrow \emptyset \cup (A \cap B')$$

$$\Rightarrow A \cap B' \Rightarrow A - B \text{ (Proved)}$$

$$\{ \therefore (A \cap B') = A - B \}$$

Let Understand with values

$$A \cup B = \{1, 2, 3, 4\}$$

$$A = \{1, 2, 3\}$$

$$B = \{1, 2, 4\}$$

$$B' = \{3\}$$

$$A - B = \{3\} \text{ — ①}$$

$$A \cap B' = \{3\} \text{ — ②}$$

from Equation ① and ②
that's proved.

Q) For all sets A, B and C
Is $(A-B) \cap (C-B) = (A \cap C) - B$?

Justify your answer?

Proof: we have $(A-B) \cap (C-B)$

$\Rightarrow$ let $x \in (A-B)$ and $x \in (C-B)$

$\Rightarrow \{ \{ x \in A \text{ and } x \notin B \} \text{ and } \{ x \in C \text{ and } x \notin B \} \}$

$\Rightarrow (x \in A \text{ and } x \in C) \text{ and } x \notin B$

$\Rightarrow (x \in A \cap C) \text{ and } x \notin B$

~~$x \in (A \cap C) - B$~~

$\Rightarrow x \in (A \cap C) - B$

So, $(A-B) \cap (C-B) = (A \cap C) - B$

★ Cartesian Product of two Set

Let A and B are two sets

then $A \times B = \{ (a, b) \mid a \in A, b \in B \}$

Example:- $A = \{ 1, 2 \}$

$B = \{ 4, 5, 6 \}$

$A \times B = \{ (1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6) \}$

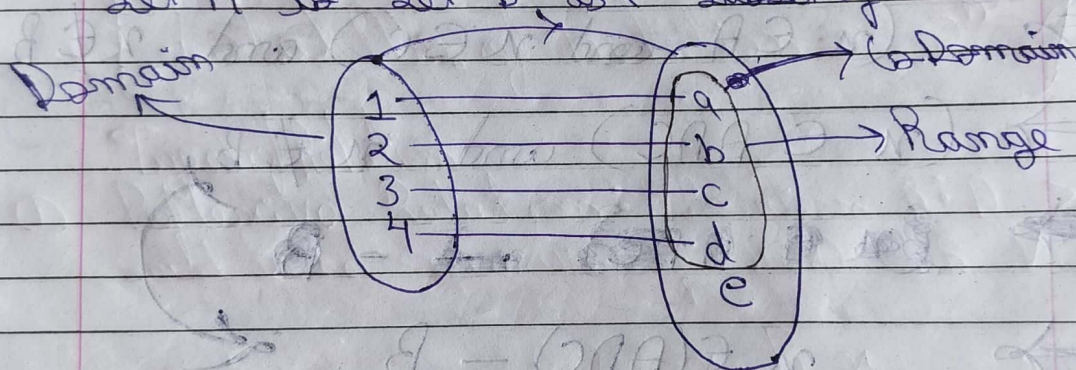


$$B \times A = \{(4,1), (4,2), (5,1), (5,2), (6,1), (6,2)\}$$

Note: Always $(A \times B) \neq (B \times A)$

It Means Cartesian Product of $A \times B$ is not equal to $B \times A$.

Relation:- Let A and B are two sets then any relation from set A to set B is a subset of $A \times B$.



$$\text{Domain} = \{1, 2, 3, 4\}$$

$$\text{Co-Domain} = \{a, b, c, d, e\}$$

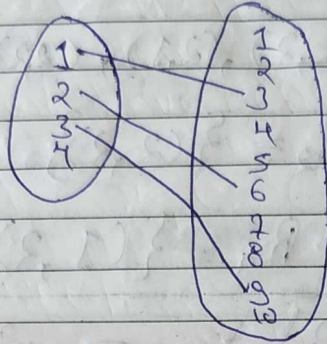
$$\text{Range} = \{a, b, c, d\}$$

Q = Let $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3, 4, \dots, 8, 9, 10\}$ and

$$R = \{(n, y) \mid y = 3n, n \in A, y \in B\}$$

Find Domain, Range and Co-Domain

$$R = \{(1,3), (2,6), (3,9)\}$$



$$\text{Domain} = \{1, 2, 3, 4\}$$

$$\text{Domain}(R) = \{1, 2, 3\}$$

$$\text{Co-Domain} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$\text{Range} = \{3, 6, 9\}$$

Types of Relation

- ① Reflexive: A Relation R is a relation from A to A ($R: A \rightarrow A$) then Relation said to be reflexive where R is Reflexive.

Ex:- If $(x, x) \in R \quad \forall x \in A$

$$A = \{1, 2, 3, 4, 5\}$$

$$A \times A = \{(1,1), (1,2), \dots, (5,4), (5,5)\}$$



Example :- $A = \{1, 2\}$

$$A \times A = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$$

$$R_1 = \{(1, 1)\}, R_2 = \{(2, 2)\},$$

$$R_3 = \{(2, 1)\}, R_4 = \{(1, 2)\}$$

$$\text{How Many Reflexive Relation} = 2^{n^2 - n}$$

$$\text{Total No. of Relation} = 2^{n^2}$$

Types of Relation

- ① Reflexive Relation: A Relation $R: A \rightarrow A$ is said to be reflexive if $(n, n) \in R \forall n \in A$

$$\text{② } n \times n$$

$$A = \{1, 2\}$$

$$A \times A = \{(1, 1)^{\textcircled{1}}, (1, 2)^{\textcircled{2}}, (2, 1)^{\textcircled{3}}, (2, 2)^{\textcircled{4}}\}$$

$$A \times A = \left\{ \begin{array}{l} (1, 1)^{\textcircled{1}}, (1, 2)^{\textcircled{2}} \\ (2, 1)^{\textcircled{3}}, (2, 2)^{\textcircled{4}} \end{array} \right\}$$

$$2^2 = 4$$

$$1 \times 2 \times 1 \times 2 = 4$$

{ 4 Relations will be formed }



Non Diagonal Elements that will be total No. of Reflexive Relation.

Date: / / Page No.:

$$B = \{1, 2, 3\}$$

$$B \times B = \left\{ \begin{array}{l} (1,1)^{\textcircled{1}}, (1,2)^{\textcircled{2}}, (1,3)^{\textcircled{3}} \\ (2,1)^{\textcircled{4}}, (2,2)^{\textcircled{5}}, (2,3)^{\textcircled{6}} \\ (3,1)^{\textcircled{7}}, (3,2)^{\textcircled{8}}, (3,3)^{\textcircled{9}} \end{array} \right\}$$

~~1 x 2 x 2 x 2 x 1 x 2 x 2 x 2 x 1~~

~~2 x 2 x 2 x 2 x 2 x 2~~

$$1 \times 2 \times 2 \times 2 \times 1 \times 2 \times 2 \times 2 \times 1 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \quad (2)$$

$$2^6 = 64$$

64 Relations will be formed.

Total Reflexive Relation formula:

$$2^{n^2-n}$$

no. of Element of A is n

$$A = n$$

$$A \times A = n^2$$

$$\text{Diagonal} = n$$

elements

$$\text{Non-Diagonal} = n^2 - n$$

Elements

$$\frac{n^2 - n}{2} = 2$$

$$\text{Total Non Reflexive Pairs / Relation} =$$

$$\text{Total Relation} - \text{Reflexive Relation}$$

$$2^{n^2} - 2^{n^2-n}$$

★ Smallest Possible Size of R.R are n

② Symmetric Relation: A Relation R on a set ' A ' is said to be Symmetric if $x R y$ then $y R x \forall x, y \in A$.

i.e. If $(x, y) \in R$ then $(y, x) \in R \forall (x, y) \in A$

$$A = \{1, 2, 3, 4\}$$

$$R_1 = \{(1, 2), (1, 3), (2, 1), (2, 3), (3, 1)\}$$

It is Symmetric but not Reflexive. (x, y) pair has (y, x) pair. It is fulfil the condition of Symmetric Relation. But not Reflexive.

$R_2 = \{ \}$ It is Symmetric Relation but not Reflexive.

★ If $\{(2, 3), (3, 3), (1, 1)\}$ ✓

★ If $\{(1, 2), (1, 3), (3, 1)\}$ ✗

③ Transitive Relation: A Relation R on a Set A is said to be Transitive if $(x R y)$ and $(y R z)$ then $(x R z) \forall x, y, z \in A$.

If $(x, y) \in R$ and $(y, z) \in R$ then

$$A \rightarrow B \quad B \rightarrow C \quad \left(\begin{array}{l} \text{Transitive} \\ \text{Relation} \end{array} \right)$$

$$(x, z) \in R.$$

$$\text{Example:- } A = \{1, 2, 3\}$$

$$R_1 = \{ \underset{(x,y)}{(1,1)}, \underset{(y,z)}{(2,2)} \} \quad \checkmark \quad \text{Transitive}$$

$$R_2 = \{ \underset{(x,y)}{(1,2)}, \underset{(y,z)}{(2,3)} \} \quad \times$$

[(x, z) की हीमा चाहिए यानि (1, 3)]

$$R_3 = \{ \} \quad \text{[always Transitive]} \quad \checkmark$$

$$\checkmark \quad R_4 = \{ \underset{(x,y)}{(1,1)}, \underset{(y,z)}{(1,2)}, \underset{(x,z)}{(2,1)} \} \quad \text{⊗}$$

$$\{ \underset{(x,y)}{(1,2)}, \underset{(y,z)}{(2,1)}, \underset{(x,z)}{(1,1)} \}$$

College Example:-

$$A = \{1, 2, 3, 4\}$$

$$\star R_1 = \{ (1,2), (2,3), (1,3) \} \quad \checkmark$$

$$\star R_2 = \{ (1,2), (2,3), (1,3), (2,4) \}$$

$$\star R_3 = \{ (1,2), (2,3), (1,3), (1,4) \}$$

$$\star R_4 = \{ (1,2), (2,1), (1,1) \} \quad \checkmark$$



④ Equivalence Relation: A Relation R on a set A is said to be equivalence if it is Reflexive, Symmetric, and Transitive.

Example: $A = \{1, 2, 3\}$

$$R_1 = \{(1,1), (2,2), (3,3)\} \quad \begin{array}{l} R \checkmark \\ S \checkmark \\ T \checkmark \\ E \checkmark \end{array}$$

$$R_2 = \{(1,1), (2,2), (3,3), (2,1), (1,2)\} \quad \begin{array}{l} R \checkmark \\ S \checkmark \\ T \checkmark \\ E \checkmark \end{array}$$

$$R_3 = \{(1,1), (2,2), (3,3), (3,2), (1,3)\} \quad \begin{array}{l} R \checkmark \\ S \times \\ T \times \\ E \times \end{array}$$

$$R_4 = \{\} \quad \begin{array}{l} R \times \\ S \checkmark \\ T \checkmark \\ E \times \end{array} \quad \text{(Not Equivalence Relation)}$$

Q = Show that $L = \{l : l \text{ is set of line}\}$ is an equivalence relation.

Solve = Proof: Let lines $l_1, l_2, l_3, \dots, l_n$

① Reflexive: $l_1 // l_1, l_2 // l_2, \dots, l_n // l_n$
Yes, it is Reflexive. $\forall l_1, l_2, l_3$

② Symmetric: $l_1 // l_2$ then $l_2 // l_1$
 $\forall l_1, l_2 \in L$
Yes, it is Symmetric

③ Transitive : $I_1 // I_2$ and $I_2 // I_3$
 then $I_1 // I_3 \quad \forall I_1, I_2, I_3 \in L$
 Yes, it is Transitive.

Resolved Yes, $L = \{I : I \text{ is set of line}\}$ is Equivalence
 by Relation of Reflexive, Symmetric and
 Transitive.

⑤ Irreflexive Relation: A Relation R on a Set ' A '
 is said to be irreflexive if $n R n \{ n \text{ is not}$
 related to $n \} \quad \forall n \in A$
 i.e. $(n, n) \notin R$

Example: $A = \{1, 2, 3\}$

$$A \times A = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$$

[Note:- Not Reflexive $\neq$ Irreflexive]

$$R_1 = \{(1, 1), (1, 2)\} \quad \text{Not Irreflexive}$$

$$R_2 = \{(1, 2), (1, 3), (2, 1)\}$$

$$R_3 = \{(1, 1), (2, 2)\} \quad (\text{Not Reflexive})$$

$$R_4 = \{\} \quad \text{Irreflexive} \quad \leftarrow (\text{Smallest Pair})$$



Note: Maximum element in Reflexive Pair / Set

$|i.e.r.| = \text{Diagonal } \times (\text{नहीं लेते हैं})$
 $\text{Non-Diagonal (Pair है)}$
 $(n^2 - n)$

$$\text{formula} = n^2 - n$$

(6) Antisymmetric Relation: A Relation R is said to be Antisymmetric if $x R y$ and $y R x$ then $x = y \forall x, y \in A$.

i.e. If $(x, y) \in R$ then $(y, x) \in R$ only

If $x = y \forall x, y \in A$.

Example:- Let $A = \{1, 2, 3\}$

$$A \times A = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$$

$$R_1 = \{(1, 2), (2, 1), (2, 1)\} \checkmark \text{ Anti-Sym-metric}$$

Note:- यदि (x, y) आया तो (y, x) नहीं आना चाहिए। यदि आया तो तो Anti-Symmetric नहीं होगा

$$R_2 = \{ (1,1), (2,2), (3,3), (1,2), (1,3), (2,3) \}$$

✓ Antisymmetric

Another Definition:

Anti Symmetric Relation: A Relation R is said to be AntiSymmetric Relation If $(x, y) \in R$ then $(y, x) \notin R$ unless $x = y$.
(जब तक)

$$\left[\begin{array}{l} \text{Total No. of} \\ \text{AntiSymmetric Relation} \end{array} \right] = 2^n \cdot 3^{\frac{n^2-n}{2}}$$

⑦ Asymmetric Relation: A Relation R is said to be Asymmetric Relation If $x R y$ and $y R x$ then $x = y \forall x, y \in A$

i.e. if $(x, y) \in R$ then $(y, x) \notin R$

$\forall x, y \in A$.

Example:- Let $A = \{1, 2, 3\}$

$$A \times A = \{ (1,1), (1,2), (1,3)$$

$$(2,1), (2,2), (2,3)$$

$$(3,1), (3,2), (3,3) \}$$



$$R_1 = \{(1, 2), (1, 3), (1, 1)\}$$

- It is not Symmetric.
- It is Antisymmetric.
- It is ~~not~~ Asymmetric.

$$R_2 = \{(1, 2), (1, 3), (2, 3)\}$$

- It is not Symmetric.
- It is Antisymmetric.
- It is Asymmetric.

$$R_3 = \{\}$$

- It is Asymmetric

• Note:-

- * Every Asymmetric is Antisymmetric.
- * Every Antisymmetric is Asymmetric.

★ Total No. of Asymmetric Relation are

Formula:- $\frac{n^2 - n}{2}$

where, n is no. of elements in A Set.

Another Definition on Asymmetric Matrix:

A Relation R on a set A is called Asymmetric if no $(y, x) \in R$ when $(x, y) \in R$.

• Partial Order Set (PO Set)

POSET, known as Partially Ordered Set, works on the Principle of Partial Ordering Relation. A Relation R is said to be Partial Ordered Relation when it can satisfy the following Properties:

1. R is Reflexive, i.e., if Set $A = \{1, 2, 3\}$ then $R = \{(1, 1), (2, 2), (3, 3)\}$ is a Reflexive Relation.
2. R is Anti-Symmetric, i.e., if R contains $(1, 2)$ then $(2, 1)$ is not allowed.
3. R is Transitive, i.e., If R contains $(1, 2), (2, 3)$, then it should contain $(1, 3)$ to make it Transitive.

Note:- Unlike Asymmetry, Anti-Symmetry allows Reflexive elements like (a, a) or (b, b) in a Relation.

Example: Let Set $A = \{1, 2, 3\}$

$$R_1 = \{(1, 1), (2, 2), (3, 3)\}$$

$$A \times A = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$$



★ $R_1 = \{(1,1), (2,2), (3,3)\}$

For Relation R_1

- Reflexive: Since $(1,1), (2,2), (3,3)$ are present so it is Reflexive.
- Anti-Symmetry: It Allows Reflexive pairs, so it is Anti-Symmetric.
- Transitive: Reflexive Pairs are Also Transitive.

★ $R_2 = \{(1,1), (2,2), (3,3), (1,2), (2,1)\}$

For Relation R_2

- Reflexive: Since $(1,1), (2,2), (3,3)$ are present so it is Reflexive.
- Anti-Symmetric: For $(1,2)$ there is $(2,1)$ so not Anti-Symmetric.
- Transitive: There are no such pairs $(a,b), (b,c)$ such that (a,c) is not present.

★ $R_3 = \{\}$

Reflexive: Null Set doesn't contain either of $(1, 1)$ $(2, 2)$ $(3, 3)$.

Therefore, R_1 is a ~~POSET~~ POSET, but

R_2 and R_3 are not.

(Reflexive Condition is not fulfilled)

Pigeon Hole Principal:

If there are more Pigeons than pigeon

then Atleast One Pigeon Hole Contains more than one pigeon.

In Another way to Explain this Statement. If you have 8 Socks in three colours, there are atleast two Socks of Same colour.

Or In another way

We can say that, If $n+1$ objects are put into n boxes, then at least one box contains two or more objects.

Principal of Mathematical Induction

Mathematical Induction is a method of proving that a statement is true for all n natural numbers or positive Integers.



It Involves two Steps:

1. Base Case: Prove that the Statement is true for the first natural number (usually $n=1$).

2. Inductive Step: Assume the Statement is true for some natural number $n=k$. Then, prove that the Statement is true for the next natural number $n=k+1$.

If Both Steps are Successful, the Statement is true for all natural numbers.

Proof / Proving the Formula for $\frac{n(n+1)}{2}$

⊙

$$\text{Proving: } 1+2+3+4+\dots+n = \frac{n(n+1)}{2}$$

$$\text{Proof: Sum of first } n \text{ natural numbers} = 1+2+3+\dots+n = \frac{n(n+1)}{2}$$

Step 1: Base Case

for $n=1$

$$1 = \frac{1(1+1)}{2} = \frac{2}{2} = 1$$

The Base Case is True.

Step 2: Inductive Step

Assume the Statement is true for $n=k$.

$$1+2+3+\dots+k = \frac{k(k+1)}{2}$$

Now, prove the Statement for $n=k+1$

$$1+2+3+\dots+k+(k+1) = \frac{k(k+1)}{2} + (k+1)$$

Let in Right Side:

$$\Rightarrow \frac{k(k+1)}{2} + (k+1)$$

$$\Rightarrow \frac{k(k+1)}{2} + \frac{2(k+1)}{2}$$

$$\Rightarrow \frac{k(k+1) + 2(k+1)}{2}$$

$$\Rightarrow \frac{(k+1)(k+2)}{2}$$

That's matches the form we want:

$$1+2+3+\dots+(k+1) = \frac{(k+1)(k+2)}{2}$$

So, $\frac{(k+1)(k+2)}{2}$ is True.

Now, By Principal of Mathematical Induction

The formula $\frac{n(n+1)}{2}$ is true for all natural numbers or Positive Integers n .



Q. $\frac{n(n+1)}{2}$ is for $\forall n \in \mathbb{N} / \mathbb{Z}^+$

Principle of Inclusion & Exclusion

The Principle of Inclusion-Exclusion (PIE) calculates the size of the union of multiple sets by adding the sizes of individual sets, subtracting the sizes of their pairwise intersections, and adding back the size of their triple intersections.

$$\text{i.e. } |A \cup B \cup C| = |A| + |B| + |C| \\ - |A \cap B| - |B \cap C| - |A \cap C| \\ + |A \cap B \cap C|$$

for example: let $A = \{a, b, c, d, e\}$

$B = \{a, b, e, g, h\}$

$C = \{b, d, e, g, h, k, m, n\}$

Solve by Principle of Inclusion and Exclusion for three sets

$$A = \{a, b, c, d, e\} \Rightarrow |A| = 5$$

$$B = \{a, b, e, g, h\} \Rightarrow |B| = 5$$

$$C = \{b, d, e, g, h, k, m, n\} \Rightarrow |C| = 8$$

$$A \cap B = \{a, b, e\} \Rightarrow |A \cap B| = 3$$

$$B \cap C = \{b, e, g, h\} \Rightarrow |B \cap C| = 4$$

$$A \cap C = \{b, d, e\} \Rightarrow |A \cap C| = 3$$

$$A \cap B \cap C = \{b, e\} \Rightarrow |A \cap B \cap C| = 2$$

Put Value in Principle of Inclusion & Exclusion.

$$|A \cup B \cup C| = 5 + 5 + 8 + 3 - 4 - 3 + 2$$

$$= 10$$

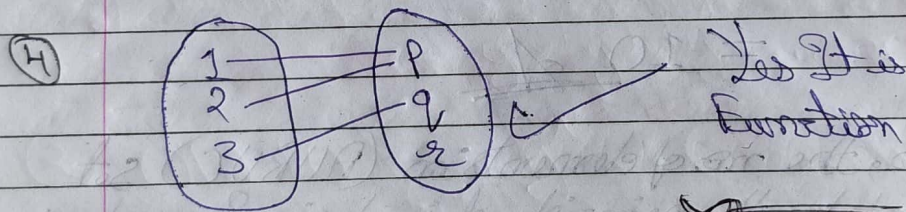
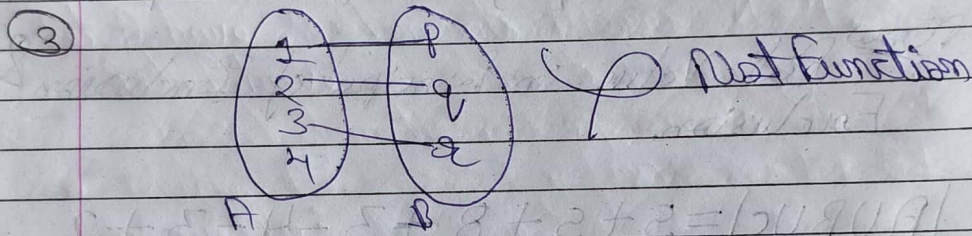
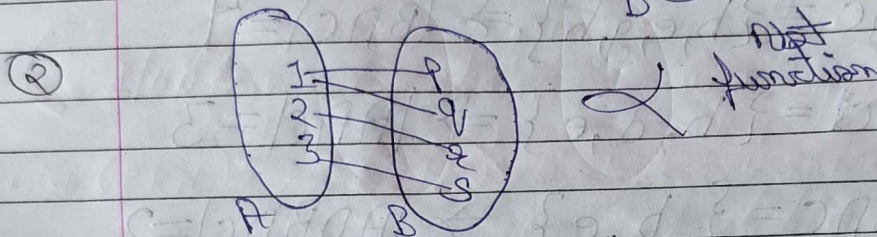
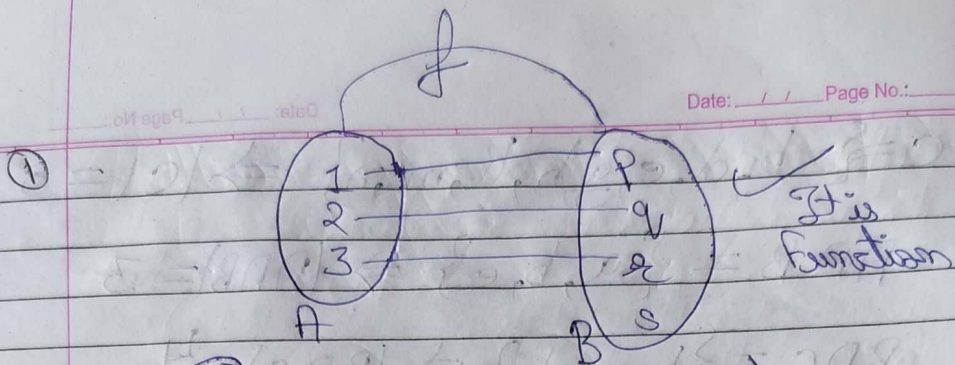
Yes, the no. of elements in $(A \cup B \cup C)$ set and by the principle of inclusion & exclusion the no. of elements are 10 by some calculation.

Function: A Relation / Mapping / Rule

Function from Set A to Set B is called a function if every element of Set A related to element of Set B has one and only one image in Set B.

(Unique Images होने चाहिए तभी function कहलाएगा)





★ Domain of Function

• If f is a function from Set A to Set B then, A is called the Domain of function f .

• The Set of all inputs for a function is called its domain.

★ Codomain of Function

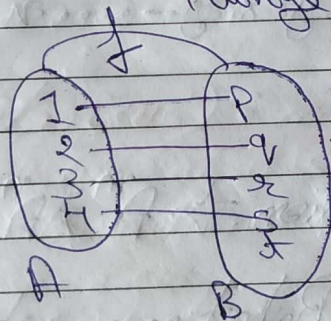
- If f is a function from Set A to Set B then, B is called the codomain of function f .

- ~~The set of all inputs for a function is called its~~ allowable
- The Set of all ~~its~~ outputs for a function is called its codomain.

★ Range of Function

The Set of all those elements of Set B which are related to Set A is called Range of a function.

Range Set of $f \subseteq B$ or



D = Domain

CD = CoDomain

R = Range

$$A = f(D) = \{1, 2, 3, 4\}$$

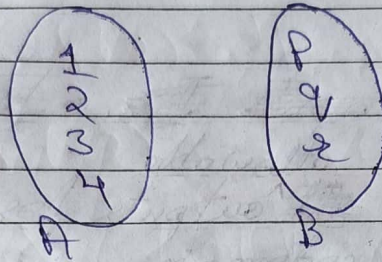
$$B = f(CD) = \{p, q, r, s, t\}$$

$$f = f(\text{Range}) = \{p, q, r, s\}$$

— ∞ —

#1 How Many Functions are Possible / Relation

#1 Example:



Set A have 4 elements Means $n=4$.

Set B have 3 elements Means $y=3$.

Total function are possible $= 3^4$

$$= 81$$

formula:

Total function are possible
 $= y \times y \times y \dots n \text{ times}$

$$= y^n$$

Total No. of Relation $\Rightarrow A^m \times A^n = A^{m \times n}$

#2 How Many Relations which are not functions.



$$2^n y - y^n$$



Kinds of Functions:

- ① one to one (or) one-one
 (or) Injective Function

A function f such that A to B ~~is~~
 $[f: A \rightarrow B]$ is called one-one function
 if no two elements in
 set A are mapped to same elements of
 set B .

$$f(x_1) = f(x_2) \\ \Rightarrow x_1 = x_2$$

① Example: Let $f: \mathbb{R} \rightarrow \mathbb{R}$ such that

$$f(x) = ax + b \quad \forall x \in \mathbb{R}$$

$$\text{let } x_1, x_2 \in \mathbb{R}$$

$$\text{and } f(x_1) = f(x_2)$$

$$\Rightarrow ax_1 + b = ax_2 + b$$

$$\Rightarrow x_1 = x_2$$

$\therefore f$ is one-one

② Example: Let $f: \mathbb{R} \rightarrow \mathbb{R}$ such that

$$f(x) = x^2 \quad \forall x \in \mathbb{R}$$



Let $-2, 2 \in \mathbb{R}$

$$f(-2) = 4$$

$$f(2) = 4$$

$$\therefore f(-2) = f(2)$$

$$\Rightarrow \text{But } -2 \neq 2$$

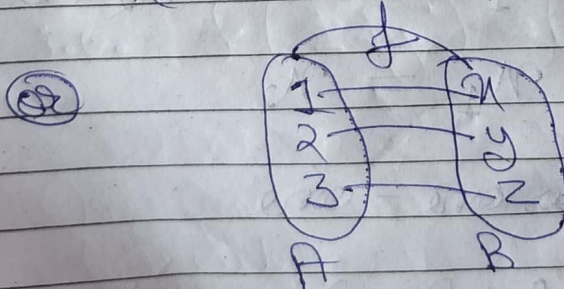
$\Rightarrow \therefore f$ is not one-one

- Define $f: \mathbb{Z} \rightarrow \mathbb{Z}$ by $f(n) = 2n + 1$ show that f is one-to-one.
- Define $f: \mathbb{Z} \rightarrow \mathbb{Z}$ by $f(n) = 3n^2 + 2$ show that f is not one-one.

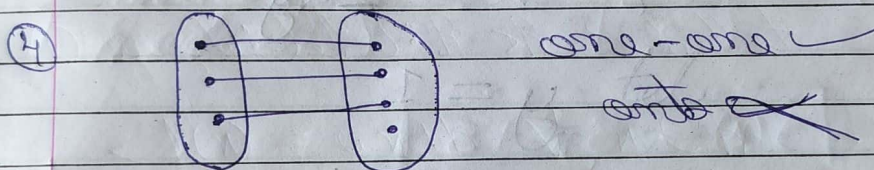
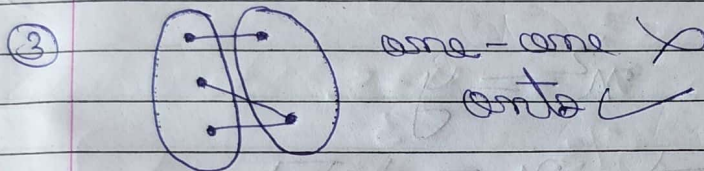
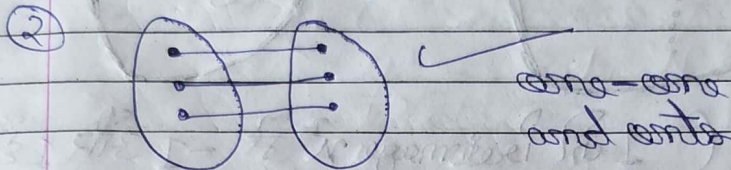
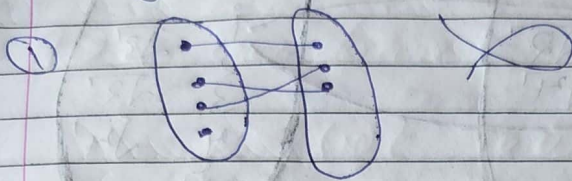
② ONTO Function: (Surjective)

A function $f: A \rightarrow B$ is said to be onto if $\forall y \in B \exists$ an Element

$x \in A$ such that $f(x) = y$



★ Recognition, the function are onto or not.



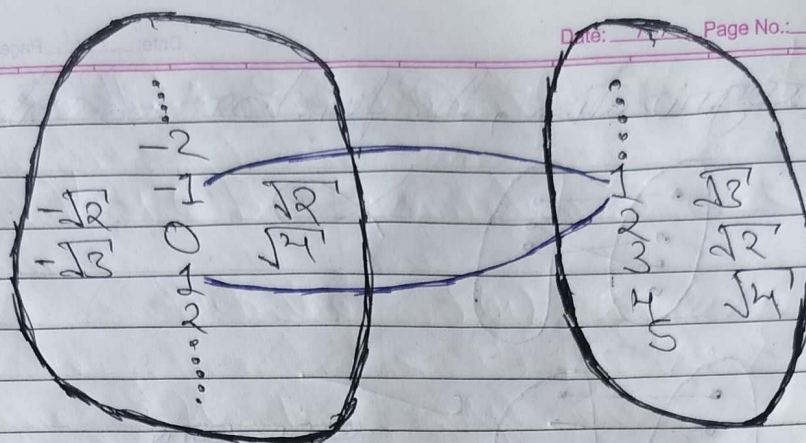
★ If f is one-one then onto.

Total Number of Function when $|A| = |B|$

formula:- n @ $n!$

Q. Let $f: \mathbb{R} \rightarrow \mathbb{R}^+$ defined by
 $f(n) = n^2 \quad \forall n \in \mathbb{R}$





(1 की Preimage x^2 में -1 और 1 है)

$$\text{Sol}^n \Rightarrow x^2 = y$$

$$\Rightarrow x = \pm \sqrt{y}$$

$$\text{let } y = 1$$

$$\Rightarrow x = \pm 1$$

$$\Rightarrow x = +1 \text{ or } x = -1$$

that's Function are onto

$$\text{i.e. } f(\pm \sqrt{y}) = (\pm \sqrt{y})^2 = y$$

③ One-One and Onto Function (Bijective)

A function f such that from Set A to B is said to be Bijective.

If it is one one & onto.

Example: Show that the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 3x^3 + 5 \quad \forall x \in \mathbb{R}$ is a bijection.

Case I: for one-one

Let $x_1, x_2 \in \mathbb{R}$ and $f(x_1) = f(x_2)$

$$f(x_1) = 3x_1^3 + 5$$

$$f(x_2) = 3x_2^3 + 5$$

$$f(x_1) = f(x_2)$$

$$3x_1^3 + 5 = 3x_2^3 + 5$$

$$x_1^3 = x_2^3$$

$$\Rightarrow x_1^3 - x_2^3 = 0$$

$$\because a^3 - b^3 = (a-b)(a^2 + b^2 + ab)$$

$$\therefore a = x_1 \text{ and } b = x_2$$

Put values

$$(x_1 - x_2)(x_1^2 + x_2^2 + x_1 x_2) = 0$$

$$\Rightarrow x_1 - x_2 = 0$$

Yes, it is One-One Function.

$\therefore f$ is one-one.



Case II: For Onto

$$\left\{ \begin{array}{l} \forall y \in B \exists \text{ an element } x \in A \\ \text{Such that } f(x) = y \end{array} \right\}$$

$$3x^3 + 5 = y$$

$$3x^3 = y - 5$$

$$x^3 = \frac{y-5}{3}$$

$$x = \left[\frac{y-5}{3} \right]^{1/3} \left\{ \begin{array}{l} \because x \text{ is real} \\ \text{Number A} \end{array} \right\}$$

$$\boxed{f(x) = 3x^3 + 5}$$

$$f(x) = 3 \left[\left[\frac{y-5}{3} \right]^{1/3} \right]^3 + 5$$

$$f(x) = 3 \left[\frac{y-5}{3} \right]^{3/3} + 5$$

$$f(x) = 3 \left(\frac{y-5}{3} \right) + 5$$

$$f(x) = y - 5 + 5$$

$$f(x) = y$$

Unit 2

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y is an element of R

Since for each $y \in R \exists$ an
(H)

$$x = \left[\frac{y-5}{3} \right]^{1/3} \in R$$

such that $f(x) = y$

$\therefore f$ is onto

Since, f is one-one and onto function

Both therefore f is Bijective / Bijection.

